

North Dakota Mathematics Talent Search 2008-2009
Problem Set 2
Problems due March 30th, 2009

1. Find all the real numbers x with the property that

$$\frac{x}{x^2 - 5x + 7}$$

is an integer.

2. If $x + \frac{1}{x} = -1$, find the value of $x^{2010} + \frac{1}{x^{2010}}$.
3. Is the following equality true?

$$\sqrt{5 + \frac{5}{24}} = 5\sqrt{\frac{5}{24}}$$

Justify your answer.

4. Let d_1, d_2, \dots, d_k be all the distinct positive divisors of the positive number n . Show that

$$(d_1 \cdot d_2 \cdot d_3 \cdot \dots \cdot d_k)^2 = n^k.$$

5. The sum of 63 positive integers is 2000. Prove that at least two of these integers are equal to each other.