North Dakota Mathematics Talent Search 2007-2008
Problem Set 3
Problems due May 31^{st}, 2008

1. Let a, b, c be odd integers. Prove that the quadratic equation $ax^2 + bx + c = 0$ does not have solutions in the set of rational numbers.

2. Prove that the product of eight consecutive positive integers cannot be a perfect square.

3. Let a, b, c, d be positive integers such that $ad = bc$. Prove that $a+b+c+d$ cannot be a prime number.

4. Let n be an integer. If $n \geq 2$, prove that $\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ cannot be an integer.

5. Find the positive integers n such that all the numbers $n+1$, $n+3$, $n+7$, $n+9$, $n+13$, and $n+15$ are prime.