

North Dakota Mathematics Talent Search 2007-2008
Problem Set 2
Problems due March 15th, 2008

1. The student lockers at Nightmare Highschool are numbered consecutively beginning with locker number 1. The plastic labels used to number the lockers cost two cents apiece. Thus, it costs two cents to label locker number 9 and four cents to label locker number 11. If it costs \$137.94 to label all the lockers, how many lockers are there at the school?
2. Find the number of ordered quadruples (x_1, x_2, x_3, x_4) of positive odd integers that satisfy $x_1 + x_2 + x_3 + x_4 = 98$.
3. In how many ways can one arrange the numbers 21, 31, 41, 51, 61, 71, and 81 such that the sum of every four consecutive numbers is divisible by 3?
4. We call a 7-digit telephone number $d_1d_2d_3-d_4d_5d_6d_7$ *memorable* if the prefix sequence $d_1d_2d_3$ is exactly the same as either of the sequences $d_4d_5d_6$ or $d_5d_6d_7$ (possibly both). Assuming that each d_i can be any of the ten decimal digits $0, 1, \dots, 9$, find the number of different memorable telephone numbers.
5. Nine chairs in a row are to be occupied by six students and Professors X, Y, and Z. These three professors arrive before the six students and decide to choose their chairs so that each professor will be between two students. In how many ways can Professors X, Y, and Z choose their chairs?