

**North Dakota Mathematics Talent Search 2007-2008**  
**Problem Set 1**  
**Problems due December 1<sup>st</sup>, 2007**

1. In how many ways can we distribute 10 books to two persons?
2. Start writing down in increasing order the positive integers beginning with 1, say 123456789101112131415161718192021.... Find the 34788<sup>th</sup> digit in this chain.
3. Consider an  $m \times n$  table with  $m$  rows and  $n$  columns whose entries consist of the integers  $+1$  and  $-1$  such that the product of the elements on each row and on each column is  $-1$ .
  - (a) Give an example of such a table for  $m = 3$  and  $n = 5$ .
  - (b) Show that if such an  $m \times n$  table exists, then  $m$  and  $n$  have the same parity.
4. How many positive integer solutions does the equation  $\frac{1}{x} + \frac{1}{y} = \frac{1}{12}$  have? (For  $x \neq y$  the solutions  $(x, y)$  and  $(y, x)$  are considered different.)
5. (a) Find the exponent of 2 in the prime factor decomposition of  $1 \cdot 2 \cdot 3 \cdot \dots \cdot 99 \cdot 100$ .  
(b) How many ending zeros does  $1 \cdot 2 \cdot 3 \cdot \dots \cdot 99 \cdot 100$  have?