

**North Dakota Mathematics Talent Search 2006-2007**  
**Problem Set 3**  
**Problems due May 15<sup>th</sup>, 2007**

1. Let  $a, b, c$  be real numbers such that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a+b+c}.$$

Prove that

$$\frac{1}{a^5} + \frac{1}{b^5} + \frac{1}{c^5} = \frac{1}{a^5 + b^5 + c^5}.$$

2. Find  $\max\{2^{237}, 3^{158}\}$ .
3. The arithmetic average of  $n$  numbers  $a_1, a_2, \dots, a_n$  is given by

$$\frac{a_1 + a_2 + \dots + a_n}{n}.$$

Assume that the arithmetic average of 50 numbers is 10. If the arithmetic average of the first 40 numbers is 12, find the arithmetic average of the last 10 numbers.

4. Assume that  $a$  and  $b$  are positive integers such that each of them can be written as a sum of two perfect squares. Show that  $ab$  can also be written as a sum of two perfect squares.
5. Let  $A$  be a set consisting of five integers. Show that  $A$  contains two distinct elements  $a$  and  $b$  such that either  $a + b$  or  $a - b$  is a multiple of 7.