

North Dakota Mathematics Talent Search 2006-2007
Problem Set 1
Problems due December 1st, 2006

1. Find the last digit of 1989^{1989} (in decimal notation).
2. Let a_1, a_2, \dots, a_{11} be eleven distinct positive integers, each less than 21. Prove that among these numbers one can find two numbers such that one of them divides the other one.
3. Can a 8×8 chessboard be covered with 1×2 dominoes such that only a1 and h8 remain uncovered?
4. A group of 40 students participated in a math contest that consists of four problems. Out of the 40 students, 25 solved the first problem, 30 students solved the second problem, 35 solved the third problem and 33 solved the fourth problem. Prove that at least 3 students solved all four problems.
5. Let $\{a_1, a_2, \dots, a_9\}$ be a permutation of the numbers from 1 to 9. Show that the product

$$(a_1 - 1)(a_2 - 2) \dots (a_9 - 9)$$

is an even number.

6. The sum of the digits of a positive integer written in decimal notation is 2006. Can this number be a perfect square?
7. Prove that the distance between any two points inside a triangle is not greater than half the perimeter of the triangle.