

**North Dakota Mathematics Talent Search 2005-2006**  
**Problem Set 3**  
**Problems due May 1<sup>st</sup>, 2006**

1. Find two disjoint subsets  $A$  and  $B$  of the set of positive rationals  $\mathbb{Q}_+^*$  such that  $A \cup B = \mathbb{Q}_+^*$  and  $B = \{xy \mid x, y \in A\}$ .
2. For each integer  $n \geq 2$ , denote by  $p(n)$  the greatest prime number less than or equal to  $n$ , and by  $q(n)$  the smallest prime number strictly greater than  $n$ .
  - (a) Prove that  $S(n) = \sum_{k=2}^n \frac{1}{p(k)q(k)} < \frac{1}{2}$ .
  - (b) Prove that  $\frac{2}{1 - 2S(2010)}$  is an integer.
3. Find all the positive integers  $n$  with the property that there exist integers  $a, b$  with  $n^2 = a + b$  and  $n^3 = a^2 + b^2$ .

Send your solutions to:

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Fargo, ND 58105

Please include:

Name:  
Address:  
High School:  
Teacher:  
GPA (optional):  
email address: