

**North Dakota Mathematics Talent Search 2005-2006**  
**Problem Set 2**  
**Problems due March 1<sup>st</sup>, 2006**

1. Find the remainder obtained when we divide  $N = 6^{102} + 6^{103} + \dots + 6^{2006}$  by 43.
2. Let  $x$  be a real number such that  $x + \frac{1}{x}$  is an integer. Prove that  $x^5 + \frac{1}{x^5}$  is also an integer.
3. Consider the polynomial  $P(X) = 3X^2 + 3mX + m^2 - 1$ , where  $m$  is a fixed real number, and let  $a, b$  be the solutions of the equation  $P(X) = 0$ . Prove that  $P(a^3) = P(b^3)$ .
4. Let  $n$  be an integer greater than 1. Prove that  $3^n + 5^n$  is not divisible by  $3^{n-1} + 5^{n-1}$ .
5. Let  $S = \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{2006^2}$ . Prove that  $S < \frac{2005}{2006}$ .

Send your solutions to:

Talent Search  
Department of Mathematics  
300 Minard Hall  
North Dakota State University  
Fargo, ND 58105

Please include:

Name:  
Address:  
High School:  
Teacher:  
GPA (optional):  
email address:

You may also submit your solutions by e-mail to [catalin.ciuperca@ndsu.edu](mailto:catalin.ciuperca@ndsu.edu).