

North Dakota Mathematics Talent Search 2005-2006
Problem Set 1
Problems due December 1st, 2005

1. Given five points inside a square of size 1×1 , prove that at least two of them are situated at a distance less than 0.75 from each other.
2. (a) The sum of $2n$ positive integers is $2n^2 + n - 1$. Prove that at least two of these integers are equal.
(b) The sum of $2n$ positive integers is $n^2 + n - 1$. Prove that at least three of these integers are equal.
3. A group of dogs and cats went to a pizza restaurant where all the pizzas had exactly 12 slices. Each dog ate 6 or 7 slices and each cat ate 2 or 3 slices. Four pizzas were not enough for them, but they could not finish the fifth one. How many dogs and cats went to the restaurant?
4. Let a, b, c be distinct nonzero real numbers. Assume that the quadratic equations $x^2 + ax + bc = 0$ and $x^2 + bx + ca = 0$ have a common root. Prove that the remaining roots satisfy the quadratic equation $x^2 + cx + ab = 0$.
5. Find all the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ that satisfy $x^2 f(x) + f(1 - x) = 2x - x^4$ for all $x \in \mathbb{R}$.

Send your solutions to:

Talent Search
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Fargo, ND 58105

Please include:

Name:
Address:
High School:
Teacher:
GPA (optional):
email address:

You may also submit your solutions by e-mail to catalin.ciuperca@ndsu.edu.