

(1) Evaluate the following limits:

(a)  $\lim_{x \rightarrow 3} \frac{x\sqrt{x+1} - 6}{x-3}$

(b)  $\lim_{t \rightarrow -\infty} \frac{2 - \sqrt{4t^2 - 1}}{3t}$

(c)  $\lim_{x \rightarrow 0} \frac{\sqrt{ax^2 + 1} - 1}{x^2}$

(d)  $\lim_{x \rightarrow \infty} \frac{\sqrt[3]{x^6 - 1}}{\sqrt{2x^4 + 3x}}$

(e)  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^3 + 3x - 14}$ .

(f)  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + ax + b} - \sqrt{x^2 + cx + d})$

(g)  $\lim_{x \rightarrow \infty} \frac{x \sin x}{x^4 + 1}$

(2) Find the derivative of the following functions:

(a)  $g(t) = \frac{8\sqrt{t}}{t^2 + 1}$

(b)  $h(x) = \frac{x^7 + x^6}{e^x + 2}$

(c)  $m(x) = (x^4 + 1)(x^2 + 1)(e^x + 7)$

(d)  $w(t) = \frac{e^t \sqrt{t}}{t^3 + t^2 + t^{-1}}$ .

(e)  $p(x) = x^2(\cos x)(\sin x - x^{13})$

(3) Let  $f(x)$  be a continuous function such that  $f(3) = 2$ . Compute  $\lim_{x \rightarrow 1} f\left(\frac{x^2 - 1}{x^2 - x}\right)$ .

- (4) Let  $f(x)$  be a differentiable function and let  $g(x) = xf(x)$ . Use the definition of derivative to show that  $g'(x) = xf'(x) + f(x)$ .
- (5) Find all horizontal and vertical asymptotes of  $f(t) = \frac{\sqrt{t^2 + 2t}}{3t}$ .
- (6) Show that the function  $z(t) = 3t + 2 \cos t$  has at least one real root.
- (7) Consider  $q(x) = \begin{cases} ax^2 + ax, & \text{if } x < 1 \\ -ax^2, & \text{if } x \geq 1 \end{cases}$
- (a) For what value(s) of  $a$ , if any, is  $q(x)$  continuous everywhere?
- (b) For what value(s) of  $a$ , if any, is  $q(x)$  differentiable everywhere?
- (8) Use the definition of derivative to compute the derivative of  $f(x) = \frac{ax}{bx + c}$ .
- (9) A cannonball is fired up in the air from the top of a building so that its height (in feet) above the ground at any time  $t$  (in feet) is given by  $s(t) = 1600 + 320t - 16t^2$ .
- (a) How tall is the building?
- (b) When does the cannonball hit the ground?
- (c) Find an equation that gives the velocity of the cannonball at any time.
- (d) How high does the cannonball go?
- (10) Let  $p(x) = \frac{f(x)g(x)h(x)}{m(x)}$ .
- (a) Find  $p'(x)$ .
- (b) Find  $p'(x)$  for  $f(x) = e^x + 1$ ,  $g(x) = 3x^4 - x^3 + 10$ ,  $h(x) = \sin x$ , and  $m(x) = e^x \cos x$ .
- (11) Use the definition of limit to prove that  $\lim_{x \rightarrow -3} 2x + 7 = 1$ .