North Dakota State University
Department of Mathematics
MATH 265 Calculus III
Instructor: Cristina Popovici

NAME (please print clearly):

SAMPLE FINAL EXAM

Question 1 (10 points):

Question 2 (10 points):

Question 3 (20 points):

Question 4 (20 points):

Question 5 (20 points):

Question 6 (20 points):

TOTAL SCORE:

Notes:
1. You have 120 minutes to complete the exam.
2. For full credit you must show your work completely. Simply writing down an answer without justifying it will receive very little partial credit.
3. NO TEXTBOOKS, NOTES or CALCULATORS are allowed while you take this exam.

1. (a)(5 points) Find the angle between the planes $x+y-z=2$ and $3x-4y+5z=6$.
(b)(5 points) Find symmetric equations for the line of intersection L of these two planes.

2. (10 points) Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + z = 2$.

3. (a)(10 points) If $f(x, y) = \sin\left(\frac{x}{1+y}\right)$ calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(b)(10 points) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by the equation

$$x^2 + y^2 + z^2 = 3xyz.$$

4. (20 points) Use a triple integral to evaluate the volume of the solid E enclosed by the paraboloid $x = y^2 + z^2$ and the plane $x = 16$.

5. (20 points) Use Stokes' Theorem to compute the integral $\iint_S \operatorname{curl} \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{S}}$, where $\overrightarrow{\mathbf{F}}(x, y, z) = xz\,\overrightarrow{\mathbf{i}} + yz\,\overrightarrow{\mathbf{j}} + xy\,\overrightarrow{\mathbf{k}}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 9$ that lies inside the cylinder $x^2 + y^2 = 4$ and above the xy-plane.

6. (20 points) Use the Divergence Theorem to evaluate $\iint_S \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{S}}$, where

$$\overrightarrow{\mathbf{F}}(x, y, z) = xy\,\overrightarrow{\mathbf{i}} + (y^2 + e^{xz^2})\,\overrightarrow{\mathbf{j}} + \sin(xy)\,\overrightarrow{\mathbf{k}}$$

and S is the surface of the region E bounded by the parabolic cylinder $z = 1 - x^2$ and the planes $z = 0,\ y = 0$, and $y + z = 2$.