MATH 793
SUMMER 2003
HOMEWORK 4

Due Friday August 8, 2003.

1. Let R be an SFT ring and $S \subseteq R$ be a multiplicatively closed subset.

   a) (5 pt) Show that R_S is SFT.

   b) (5 pt) Show that any homomorphic image of R is SFT.

2. Consider the domain $R = A + xB[x]$ where $A \subseteq B$ are domains. Prove the following.

   a) (5 pt) R is integrally closed if and only if B is integrally closed and A is integrally closed in B.

   b) (5 pt) R is completely integrally closed if and only if B is completely integrally closed and $A = B$.

   c) (5 pt) R is a UFD if and only if $A = B$ and B is a UFD.

3. (5 pt) Let R be a ring such that in every homomorphic image of R, every element is either a unit or a zero-divisor. Show that R is 0-dimensional.

4. (5 pt) Let R be a Noetherian ring and M a finitely-generated $R-$module. Show that any submodule of M is finitely generated.