

Math 446–646

Homework Assignment for Spring-Break (Due on March 21)

- (1) Let (\mathbb{R}, T_u) be \mathbb{R} with the usual topology. For each one of the following subsets of \mathbb{R} , determine if the relative topology and the order topology on the subset coincide.
- (a) $A = (1, 2)$
 - (b) $B = (0, 1) \cup [2, 3)$
 - (c) $[0, 1] \cup [2, 3]$
 - (d) $(0, 1) \cup \{2\}$
- (2) Let (X, T) be a topological space. Let Y be a totally ordered set and let $T_<$ be the order topology on Y . Consider two continuous functions $f : (X, T) \rightarrow (Y, T_<)$ and $g : (X, T) \rightarrow (Y, T_<)$.
- (a) Prove that the set $\{x \in X \mid f(x) \leq g(x)\}$ is closed in X . (*Hint: prove that its complement is open*).
 - (b) Let $h : (X, T) \rightarrow (Y, T_<)$ the function defined by $h(x) = \min f(x), g(x)$. Prove that h is continuous. (*Hint: use (a) and the pasting lemma*).
- (3) Prove the following properties
- (a) $\overline{A \times B} = \overline{A} \times \overline{B}$
 - (b) $(A \times B)^\circ = A^\circ \times B^\circ$
 - (c) $\overline{A \cup B} = \overline{A} \cup \overline{B}$
 - (d) $A^\circ \cup B^\circ \subset (A \cup B)^\circ$
 - (e) Find an example that shows that the inclusion $(A \cup B)^\circ \subset A^\circ \cup B^\circ$ is false.
- (4) (a) Let (X, T) be topological space that has property T_1 . Prove that if X is a finite set, then T is the discrete topology.
- (b) Find an example of a topological space (X, T) that has the Hausdorff property (and therefore, it has property T_1) such that X is a countable set and T is not the discrete topology.
- (5) Prove that a topological space (X, T) has the Hausdorff property if and only if the set $\Delta = \{(x, x) \mid x \in X\}$ is closed in $(X \times X, T \times T)$.